Calculation of Imaging Properties of Metamaterials

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Metamaterials have extraordinary optical properties, which can be utilized in novel imaging systems. Their imaging properties, however, cannot be determined by simple geometrical optics. Here we propose a method that combines full wave simulation, transfer matrix method and geometrical optics to efficiently simulate imaging systems containing metamaterials. The dispersion relation of a periodic metamaterial slab is retrieved from the S-parameters obtained from full wave simulation of the unit cell. The dispersion relation can be connected with numerical field solvers, e.g. the transfer matrix method, to calculate the field produced by a point source placed behind a metamaterial slab. Finally, light rays are derived from the full wave solution to obtain the virtual image of the point source. From the virtual distance an effective geometrical refractive index is determined, which characterizes the imaging properties of the metamaterial. Using the developed procedure, imaging possibilities with fishnet metamaterials are investigated.

***Index Terms*—Geometrical optics, Optical Imaging, Optical Metamaterials, Optical Propagation**

I. Introduction

Metamaterials are artificial structures with sub-wavelength feature sizes, which offer possibility to engineer materials with nearly arbitrary optical properties, like negative, near-zero or ultrahigh refractive index in various frequency regimes [1]. Although optical metamaterials are not commercialized yet, they are intensively researched [2]-[3] and expected to open up new possibilities for optical devices.

Applying the laws of geometrical optics, the image produced by an imaging device can be conveniently calculated with ray tracing. However, the usual ray tracing algorithms can be applied only for configurations, where the materials are homogeneous, isotropic and thick compared to the wavelength [4]. Metamaterials are mostly anisotropic, lossy and thin compared to the wavelength, hence only full wave simulation can provide an accurate image. Full wave simulation needs, however, huge computational effort as the unit cell and feature sizes of the metamaterials are smaller than the wavelength, while the full size of the imaging system is usually large compared to the wavelength.

In this paper we propose a method that efficiently combines geometrical optics, full wave simulation and the transfer matrix method to provide a fast calculation procedure for imaging systems with metamaterials.

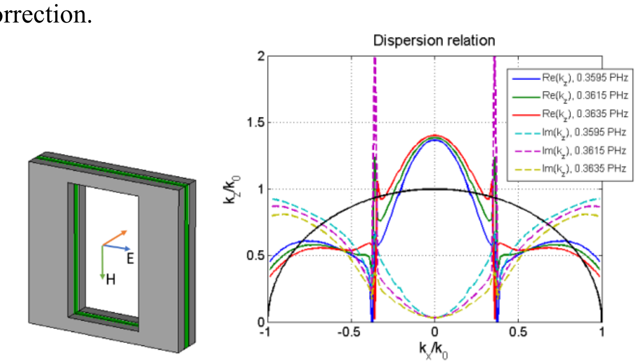
II. Effective Parameter Retrieval

Due to their sub-wavelength feature sizes metamaterials can be characterized with macroscopic material parameters in a limited frequency range. These parameters can be retrieved from the complex reflection-transmission coefficients of a finite slab [5]-[6]. The effective material parameters depend strongly on the angle of incidence. Therefore, to fully characterize the electromagnetic behavior of metamaterial it is necessary to retrieve the effective parameters for a number of incident angles, for both TE and TM polarization, similarly to natural anisotropic materials.

Full wave simulation of one metamaterial unit cell is performed with Bloch boundary condition to calculate the S-parameters for plane waves with different incident angles. These calculations do not need high computational effort as only one cell is considered.

From the S-parameters the normal wave number (k_z) is retrieved according to [6], while the lateral wave number (k_x), which is conserved at the interface of two media, can be determined from the incident angle. The generalized wave impedance (ξ) is also retrieved for each angle of incidence [6]. The relation between k_z and k_x gives the dispersion relation for the given frequency, while the generalized wave impedance can be used to calculate reflections on the boundary of the metamaterial.

Fig.1. shows the dispersion relation of a fishnet structure operating in the optical regime [7]. The size of the rectangular unit cell is 600x600 nm, the size of the window is 284x500nm, the thickness of the MgF_2 separation layer is 30nm and the thickness of the silver layers is 45nm. The structure shows an elliptic-like dispersion relation for small incident angles around 360THz, which varies rapidly with the frequency, where the fishnet can be applied e.g. for chromatic aberration correction.



(a) (b)

Fig. 1. a) The unit cell of the examined fishnet metamaterial. b) The dispersion relation of the fishnet structure (red, green, blue curves), compared to that of vacuum (black curve). k_0 is the free space wavenumber.

III. Calculation of Imaging a Point Source with a Metamaterial Slab

To calculate the image of a point source placed behind a metamaterial slab, where the length of the metamaterial is much larger than the wavelength, is a cumbersome problem and requires full wave simulation of the whole system including an extended area of the metamaterial slab. To avoid this huge computational effort, we utilize the transfer matrix method [2], with the dispersion relation presented in Fig. 1. Therefore the validity of the calculations is extended even to frequency ranges, where homogenization fails.

The electromagnetic field of the point source is decomposed into plane waves using Fourier transform. The complex amplitude of each plane wave is then calculated in the image plane behind the metamaterial slab by applying the transfer matrix method. The transfer function of the medium between the source plane and the metamaterial slab is

$$T(k_x) = \exp(i k_z^s d_s), \tag{1}$$

where d_s is the distance from the source to the metamaterial and k_z^s is the normal wave number corresponding to the surrounding medium. Similar expression can be obtained for the transfer function between the metamaterial slab and the image plane. The transfer function of a homogeneous slab with finite thickness d is

$$T(k_x) = \frac{4}{\left(\frac{\xi}{k_z^s}+1\right)\left(\frac{k_z^s}{\xi}+1\right)\exp(-i\, k_z d) + \left(\frac{\xi}{k_z^s}-1\right)\left(\frac{k_z^s}{\xi}-1\right)\exp(i\, k_z d)}, \tag{2}$$

where ξ is the generalized wave impedance and k_z is the normal wave number in the metamaterial. If internal reflections within the metamaterial are neglected, the transfer function is simplified to

$$T_{slab,2}(k_x) = (1-\Gamma_1)\exp(i k_z d)\,(1-\Gamma_2), \tag{3}$$

where Γ_1 and Γ_2 are the reflection coefficients on the first (air-slab) and second (slab-air) boundary

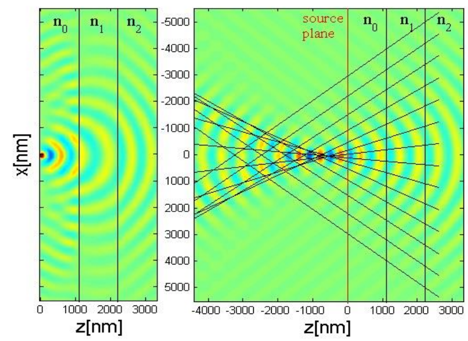
$$\Gamma_1 = (k_z^s - \xi)/(k_z^s + \xi)\,,\ \Gamma_2 = (\xi - k_z^s)/(\xi + k_z^s). \tag{4}$$

The total transfer function of the imaging system, where the metamaterial slab is surrounded with air is

$$T_{total}(k_x) = T_{air,1}(k_x) T_{slab}(k_x) T_{air,2}(k_x)\,. \tag{5}$$

The electromagnetic field distribution in the image plane is obtained by multiplying (5) by the complex amplitudes of the plane waves in the source plane. The contributions of each plane wave is summed up to provide the full electromagnetic field distribution in the image plane.

If the electromagnetic field is known in the image plane, then the virtual image of the point source, which can be observed by an observer, can be calculated. The image is perceived by the observer as if rays were propagating through vacuum from a virtual source, hence the position of this virtual source can be determined by applying the inverse transfer function of vacuum on the Fourier transform of the image plane. Fig. 2.b depicts the reversed field. The virtual source can be found at the intensity maxima.



(a) (b)

Fig. 2. The electric field distribution of a point source located near a metamaterial slab (a). The calculated virtual image of the point source after applying backward calculations from the image plane (b).

Another way to determine the position of the virtual source is the tracing back of Poynting vectors. Poynting vectors do not bend in vacuum, hence the virtual source can be found at the intersection of the straight lines elongating the Poynting vectors, just like in case of geometrical optics, as shown in Fig. 2.b. Note that the virtual distance depends on the angle of observation.

From the computed virtual distance an effective geometrical refractive index can be obtained for each incident angle α

$$n_{eff}(\alpha) = \sqrt{1/\left(\frac{(x+d)^2}{d^2}\cos^2\alpha + \sin^2\alpha\right)}\,, \tag{6}$$

where x is the distance between the real and the virtual position of the point source. The effective geometrical index allows the calculation of refraction by applying Snell's law.

IV. Further Investigations

In the full paper inhomogeneous metamaterial slabs will be discussed, and the developed procedure will be applied to determine the imaging properties of Fishnet metamaterials.

V. Acknowledgement

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